Generalized nonlinear Schrödinger equation and ultraslow optical solitons in a cold four-state atomic system

Chao Hang,[1] Guoxiang Huang,[1,*] and L. Deng[2]
[1]Department of Physics, East China Normal University, Shanghai 200062, China
[2]Electron and Optical Physics Division, NIST, Gaithersburg, Maryland 20899, USA
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We investigate the influence of high-order dispersion and nonlinearity on the propagation of ultraslow optical solitons in a lifetime broadened four-state atomic system under a Raman excitation. Using a standard method of multiple-scales we derive a generalized nonlinear Schrödinger equation and show that for realistic physical parameters and at the pulse duration of 10^{-6} s, the effects of third-order linear dispersion, nonlinear dispersion, and delay in nonlinear refractive index can be significant and may not be considered as perturbations. We provide exact soliton solutions for the generalized nonlinear Schrödinger equation and demonstrate that optical solitons obtained may still have ultraslow propagating velocity. Numerical simulations on the stability and interaction of these ultraslow optical solitons in the presence of linear and differential absorptions are also presented.

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I. INTRODUCTION

Electromagnetic wave propagation in highly resonant media has been a topic of much interest for a long time. Important research achievements in the past few decades include self-induced transparency (SIT) in two-level atoms [1,2], optical simultons in multilevel media [3,4], lasing without inversion [5], phaseonium [6], normal-mode [7], and electromagnetically induced transparency (EIT) [8]. In SIT, an optical field with sufficiently short pulse duration and high light intensity is required so that the attenuation due to spontaneous radiation of atoms can be neglected. However, a transparency mechanism under an electromagnetically induced transparency (EIT) scheme is quite different. In a typical three-state Λ-type EIT scheme a quantum interference effect induced by a control field makes the excited state of atoms a dark state, and hence a weak probe field can propagate with very low loss. It has been demonstrated that the wave propagation under EIT configuration can possess many striking features [9–16]. One of them is significant reduction of group velocity [17,18], which may lead to many new physical effects and have important technical applications [19]. Another feature is that Kerr nonlinearity of optical media can be largely enhanced through a cross-phase modulation effect. This technique has been proposed for achieving a large nonlinear phase shift [9,11] and some other nonlinear optical processes under weak driving conditions. Based on the resonantly large nonlinearity enhancement, the low absorption, and the ultraslow propagation property [8–21], it has been shown recently [22–24] that it is possible to produce a new type of optical soliton, i.e., *ultraslow optical soliton* (USOS), in highly resonant optical media. Because of their robust nature and ultraslow propagating velocity, the USOS may have the potential to be a promising candidate of well-characterized, distortion-free optical pulses and hence has important technological applications in optical and telecommunication engineering.

In Ref. [22], the USOS in a four-level system is studied using a weak nonlinear analysis on Maxell-Schrödinger (MS) equations that govern the evolution of optical field and atomic amplitudes. A nonlinear Schrödinger (NLS) equation governing the propagation dynamics of envelope of the optical field based on the assumptions of negligibly small higher order contributions is derived. In doing so, all time derivative terms in calculating the nonlinear coupling terms are neglected. It has been shown [24], however, under certain circumstances these higher order terms can have significant effects to the propagation dynamics, resulting in further group velocity correction, center frequency shift, and radiations of the USOS, etc. Thus it is important to reexamine the USOS in the four-state N-type scheme studied and investigate the corresponding corrections to the much simplified treatment.

In this work, we study soliton propagation in a four-state N-type scheme. The motivation of the present study is stimulated by the investigation of the stability of USOS reported in Ref. [22] as a function of the pulse duration of the input optical fields. Indeed, we have found that the ultraslow optical soliton becomes unstable when the pulse duration is shortened with other parameters unchanged. This is suggestive that the stability of slow or ultraslow optical solitons are very sensitive to the nonadiabatic perturbations. By using a standard method of multiple-scales we first derive a generalized NLS equation which includes the corrections due to high-order dispersion and nonlinearity of the system. We show that, for the pulse duration of 10^{-5} s as used in Ref. [22], these correction terms are indeed not significant. However, for the pulse duration of 10^{-6} or less and under the same driving conditions, we show that these corrections become significant and may not be treated simply as perturbations of the NLS equation. Thus, in order to be able to correctly predict the formation and stable propagation of slow and ultraslow optical solitons of shorter pulse durations, the

*Electronic address: gxhuang@phy.ecnu.edu.cn

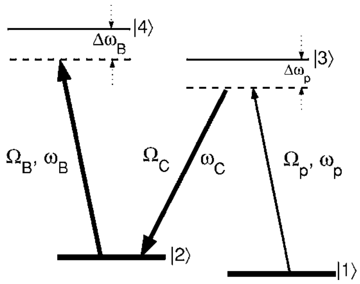

FIG. 1. The energy-level diagram and excitation scheme of a lifetime broadened four-state atomic system interacting with a weak, pulsed probe field of center frequency $\omega_p/(2\pi)$ and two strong, cw control fields of frequencies $\omega_B/(2\pi)$ and $\omega_C/(2\pi)$.

generalized NLS equation must be used. We further present soliton solutions of the generalized NLS equation derived and demonstrate that these optical solitons are stable and can propagate with very slow group velocities [25]. The paper is arranged as follows. Section II describes the four-level N-scheme. A generalized NLS equation is derived in Sec. III. In Sec. IV the soliton solutions of the high-order NLS equation are given and the physical properties of these shape-preserving fields are discussed. In Sec. V, we present numerical simulations that test the stability and interaction of USOSs in the presence of high-order linear and differential absorptions. Finally, Sec. V contains discussions and conclusion of our results.

II. THE MODEL

We start with a lifetime broadened four-state atomic system which interacts with a weak, pulsed probe field (pulse duration τ_0) of center frequency $\omega_p/(2\pi)$ (coupling $|1\rangle\to|3\rangle$ transition) and two strong, continuous-wave (cw) control fields of frequencies $\omega_B/(2\pi)$ ($|2\rangle\to|4\rangle$ transition) and $\omega_C/(2\pi)$ ($|3\rangle\to|2\rangle$ transition), respectively (see Fig. 1). The electric-field vector polarized in the x direction can be written as $\mathbf{E}=\hat{\mathbf{x}}[\mathcal{E}_p(x,y,z,t)e^{i(k_pz-\omega_pt)}+\mathcal{E}_Be^{i(k_Bz-\omega_Bt)}+\mathcal{E}_Ce^{i(k_Cz-\omega_Ct)}+\text{c.c.}]$, where $\hat{\mathbf{x}}$ is the unit vector in the x-direction and $k_l=n_l\omega_l/c$ $(l=p,B,C)$; n_l is the background index of refraction at the frequency ω_l. The MS equations governing the motion of atomic state amplitudes and time-dependent probe field are [22]

$$\frac{\partial A_1}{\partial t}=i\Omega_p^*A_3, \tag{1a}$$

$$\frac{\partial A_2}{\partial t}=-\gamma_2A_2+i\Omega_C^*A_3+i\Omega_B^*A_4, \tag{1b}$$

$$\frac{\partial A_3}{\partial t}=-(i\Delta_p+\gamma_3)A_3+i\Omega_CA_2+i\Omega_pA_1, \tag{1c}$$

$$\frac{\partial A_4}{\partial t}=-(i\Delta_B+\gamma_4)A_4+i\Omega_BA_2, \tag{1d}$$

$$\left(\frac{\partial}{\partial z}+\frac{1}{c}\frac{\partial}{\partial t}\right)\Omega_p-i\frac{c}{2\omega_p}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\Omega_p=i\kappa_{13}A_3A_1^*, \tag{1e}$$

where A_j is the probability amplitude of the atomic state $|j\rangle$ $(j=1-4)$ satisfying the conservation condition $\Sigma_{l=1}^4|A_l|^2=1$. $2\Omega_p$, $2\Omega_B$, and $2\Omega_C$ are the Rabi frequencies for corresponding transitions. $\Gamma_i=2\gamma_i$ is the decay rate of level $|i\rangle$ $(i=2,3,4)$. $\kappa_{13}=2\pi N_a\omega_p|D_{13}|/(\hbar c)$ with N_a being the particle number density and D_{31} being the dipole moment for the transition from $|1\rangle\to|3\rangle$. In deriving Eqs. (1a)–(1e) we have used the slowly varying envelope approximation for the pulsed probe field and defined two one-photon detuning $\Delta_B=\omega_{42}-\omega_B$ and $\Delta_p=\omega_{31}-\omega_p$. Additionally, we have assumed $\Delta_{21}=\omega_{21}+\omega_C-\omega_p=0$, which means that a two-photon resonance is always maintained.

Before solving the MS Eqs. (1a)–(1e) by using an approach based on the method of multiple-scales (see Sec. III), we first examine the linear properties of the system. These linear properties are the main contributors to pulsed spreading and attenuation. To achieve this, we assume that the probe field is weak so that the atomic ground state $|1\rangle$ is not depleted, i.e., $A_1\approx1$. In this case one can make a linear analysis on Eqs. (1a)–(1e). Taking Ω_p and A_j $(j=2,3,4)$ as being proportional to $\exp\{i[k(\omega)z-\omega t]\}$, one can easily get the linear dispersion relation of the system, which is given by Ref. [22] $k(\omega)=\omega/c-\kappa_{13}D_p(\omega)/D(\omega)$, where $D_p(\omega)=|\Omega_B|^2-(\omega+i\gamma_2)(\omega-\Delta_B+i\gamma_4)$ and $D(\omega)=|\Omega_B|^2(\omega-\Delta_p+i\gamma_3)+|\Omega_C|^2(\omega-\Delta_B+i\gamma_4)-(\omega+i\gamma_2)(\omega-\Delta_p+i\gamma_3)(\omega-\Delta_B+i\gamma_4)$ [26]. A general linear solution for Ω_p and A_j $(j=2,3,4)$ can be obtained by Fourier superimposing different frequency components. The group velocity and group-velocity dispersion of a pulsed probe field can be obtained by Taylor-expanding the linear dispersion relation $k(\omega)$ around the center frequency ω_p [27], giving $k(\omega)=K_0+K_1\omega+K_2\omega^2/2+\cdots$ with $K_j=[\partial^jk(\omega)/\partial\omega^j]|_{\omega=0}(j=0,1,2,\ldots)$. Here, $K_0=\phi+i\alpha/2$ gives the phase shift ϕ per unit length and absorption coefficient α of the probe field, $1/K_1$ is related to the group velocity, and $1/K_2$ describes the group velocity dispersion (i.e., pulse spreading). For probe field with a Gaussian input form, i.e., $\Omega_p(0,t)=\Omega_p(0,0)\exp(-t^2/\tau_0^2)$, we have [28]

$$\Omega_p(z,t)=\frac{\Omega_p(0,0)}{\sqrt{b_1(z)-ib_2(z)}}\exp\left[iK_0z-\frac{(K_1z-t)^2}{[b_1(z)-ib_2(z)]\tau_0^2}\right],$$

where $b_1(z)=1+2z\,\mathrm{Im}(K_2)/\tau_0^2$ and $b_2(z)=2z\,\mathrm{Re}(K_2)/\tau_0^2$. The above equation clearly shows that dispersion and dissipation effects contribute to the probe field spreading and attenuation, as expected.

III. ASYMPTOTIC EXPANSION AND A HIGH-ORDER NLS EQUATION

In this section, we apply a perturbation theory to solve Eqs. (1a)–(1e) and search for the formation and propagation of a shape-preserving probe pulse in the four-state system. We first note that nonvanishing one-photon detuning is necessary not only to introduce group-velocity dispersion but also to induce self-phase (when $\Delta\omega_p\neq0$) and cross-phase (when $\Delta\omega_B \neq 0$) modulation effects which can provide effective means to balance the detrimental dispersion effect, leading to stable formation and propagation of solitons [29]. To make the nonlinear effect in the system significant, one should increase the intensity of the probe field and hence the population of the ground state $|1\rangle$ will be depleted. To get a quantitative description of the formation and dynamics of an ultraslow optical soliton in the system, as a first step we derive a nonlinear envelope equation that describes the evolution of the probe field envelope by employing a standard method of multiple-scales [30]. We first make the following asymptotic expansion $A_j=\sum_{n=0}^{\infty}\epsilon^n A_j^{(n)}(j=1,2,3,4)$ and $\Omega_p=\sum_{n=1}^{\infty}\epsilon^n\Omega_p^{(n)}$, where ϵ is a small parameter characterizing the small population depletion of the ground state. To obtain a divergence-free expansion, all quantities on the right-hand side of asymptotic expansion are considered as functions of the multiscale variables $z_l=\epsilon^l z$ $(l=0-3)$, $t_l=\epsilon^l t$ $(l=0,1)$, $x_1=\epsilon x$, and $y_1=\epsilon y$. Substituting the expansions and the multiscale variables into Eqs. (1a)–(1e), we obtain a chain of linear but inhomogeneous equations on $A_j^{(n)}$ and $\Omega_p^{(n)}$, which can be solved order by order.

To the leading order, $O(\epsilon)$, the solution of Eqs. (1a)–(1e) is just that obtained in the linear regime, described in the last section. Since we are interested in the evolution of a pulsed probe field we take $\Omega_p^{(1)}=F\exp(i\theta)$, where $\theta=k(\omega)z_0-\omega t_0$ and F is a yet to be determined envelope function depending on the slow variables x_1, y_1, t_1 and z_j $(j=1,2,3)$.

To the second order, $O(\epsilon^2)$, a divergence-free solution requires one set

$$i\left(\frac{\partial F}{\partial z_1}+\frac{1}{V_g}\frac{\partial F}{\partial t_1}\right)=0, \tag{2}$$

where $V_g=1/K_1$ is the group velocity of the envelope F.

To the third order, $O(\epsilon^3)$, the solvability condition yields the NLS equation

$$i\frac{\partial F}{\partial z_2}-\frac{K_2}{2}\frac{\partial^2 F}{\partial t_1^2}+\frac{c}{2\omega_p}\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial y_1^2}\right)F - W\exp(-\alpha_1 z_2)F|F|^2=0, \tag{3}$$

with $\alpha_1=\epsilon^{-2}\alpha$, and

$$W=-\frac{\kappa_{13}D_p[|D_p|^2+|\Omega_C|^2(|\omega-\Delta_B+i\gamma_4|^2+|\Omega_B|^2)]}{D|D|^2}. \tag{4}$$

Equation (3) without the diffraction term [i.e., the third term on the left-hand side (lhs)] was obtained in Ref. [22] using an approach that relies on the zeroth order Taylor expansion of the nonlinear term. It has been shown [24] that such a simple technique is often oversimplified when treating highly resonant systems. Indeed, many interesting effects and phenomena can and will raise from the contributions not included in such a simple treatment.

Our goal is to study the influence of high-order nonlinear and dispersion effects on the evolution of a pulsed optical field. For this purpose we must go beyond the NLS approximation, i.e., Eq. (3), which is a third order equation. The fourth order equations, i.e., $O(\epsilon^4)$, can be obtained from the asymptotic expansion using the solutions of the first, second, and third order equations. By a detailed calculation, we get the solvability condition for fourth order equations

$$i\frac{\partial F}{\partial z_3}-i\frac{K_3}{6}\frac{\partial^3 F}{\partial t_1^3}-i\beta_1\exp(-\alpha_1 z_2)\frac{\partial}{\partial t_1}(|F|^2F) + i\beta_2\exp(-\alpha_1 z_2)\frac{\partial|F|^2}{\partial t_1}F=0, \tag{5}$$

where

$$\beta_1=-\kappa_{13}\frac{D_p}{D}(q_1+q_1^*) - \frac{D_p^2+|\Omega_C|^2[(\omega-\Delta_B+i\gamma_4)^2+|\Omega_B|^2]}{D_pD}W,$$

$$\beta_2=-\kappa_{13}\frac{D_p}{D}(2q_1+q_1^*) - \frac{D_p^2+|\Omega_C|^2[(\omega-\Delta_B+i\gamma_4)^2+|\Omega_B|^2]}{2D_pD}W,$$

with

$$\begin{aligned}q_1=&-\frac{1}{\kappa_{13}}\frac{\omega-\Delta_B+i\gamma_4}{D}\left[(\omega-\Delta_p+i\gamma_3)\left(\frac{1}{V_g}-\frac{1}{c}\right)+\left(k-\frac{\omega}{c}\right)\right]^*+\frac{1}{\kappa_{13}}\frac{D_p}{D}\left(\frac{1}{V_g}-\frac{1}{c}\right)^*+\frac{1}{\kappa_{13}D}\left\{-(\omega+i\gamma_2)\right.\\&\times\left[(\omega-\Delta_p+i\gamma_3)\left(\frac{1}{V_g}-\frac{1}{c}\right)+\left(k-\frac{\omega}{c}\right)\right]\\&\left.+|\Omega_C|^2\left(\frac{1}{V_g}-\frac{1}{c}\right)-\kappa_{13}\frac{|\Omega_C|^2(\omega-\Delta_B+i\gamma_4)}{D}\right\}^*,\end{aligned}$$

where the asterisk represents a complex conjugate.

Combining Eqs. (2), (3), and (5), we obtain

$$\begin{aligned}&i\frac{\partial F}{\partial\zeta}-\frac{K_2}{2}\frac{\partial^2 F}{\partial\eta^2}+\frac{c}{2\omega_p}\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial y_1^2}\right)F-W\exp(-\alpha_1 z_2)F|F|^2\\&+i\left[-\frac{K_3}{6}\frac{\partial^3 F}{\partial\eta^3}-\beta_1\exp(-\alpha_1 z_2)\frac{\partial}{\partial\eta}(|F|^2F)+\beta_2\exp(-\alpha_1 z_2)\frac{\partial|F|^2}{\partial\eta}F\right]=0,\end{aligned} \tag{6}$$

where $\zeta=z_2=\epsilon^2 z$ and $\eta=t_1-z_1/V_g=\epsilon(t-z/V_g)$. Equation (6) is a generalized NLS equation including diffraction, third-order linear dispersion and noninstantaneous Kerr nonlinearity. Such an equation also appears in the study of pulse propagation in nonlinear optical fibers [33].

IV. ULTRASLOW OPTICAL SOLITON SOLUTIONS OF THE GENERALIZED NLS EQUATION

The generalized NLS equation derived in the last section has complex coefficients and hence is generally not integrable. However, if a realistic set of parameters can be found so that the imaginary part of these coefficients can be made small in comparison with the corresponding real parts, then it is possible to get a shape-preserving, localized solution that can propagate for an extended distance without significant attenuation and distortion. In this case Eq. (6), when returning to original variables, reduces to ($Z_r=\mathrm{Re}[Z]$ where Z is a general symbol that represents $\widetilde{W}$, K, $\widetilde{\beta}$, etc.)

$$i\left(\frac{\partial}{\partial z}+K_{0i}\right)U-\frac{K_{2r}}{2}\frac{\partial^2 U}{\partial\tau^2}+\frac{c}{2\omega_p}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)U-\widetilde{W}_r|U|^2U$$
$$+i\left[-\frac{K_{3r}}{6}\frac{\partial^3 U}{\partial\tau^3}-\widetilde{\beta}_{1r}\frac{\partial}{\partial\tau}(|U|^2U)+\widetilde{\beta}_{2r}\frac{\partial|U|^2}{\partial\tau}U\right]$$
$$-K_{1i}\frac{\partial U}{\partial\tau}=0, \tag{7}$$

where we have set $\omega=0$ [31], $\tau=t-K_{1r}z=t-z/V_{gr}$, and $\Omega_p=U\exp(iK_{0r}z)$. The explicit expressions of the coefficients of Eq. (7) have been given in the Appendix.

For convenience of the following discussions, we write Eq. (7) into the following dimensionless form:

$$i\frac{\partial u}{\partial s}+\frac{\partial^2 u}{\partial\sigma^2}+2|u|^2u$$
$$=-i\left[d_0u-d_1\frac{\partial|u|^2u}{\partial\sigma}-d_2u\frac{\partial|u|^2}{\partial\sigma}-d_3\frac{\partial^3 u}{\partial\sigma^3}\right]$$
$$+d_4\frac{\partial u}{\partial\sigma}-d_5\left(\frac{\partial^2 u}{\partial x'^2}+\frac{\partial^2 u}{\partial y'^2}\right), \tag{8}$$

where $s=-z/(2L_D)$, $\sigma=\tau/\tau_0$, $(x',y')=(x,y)/R_\perp$, and $u=U/U_0$, $L_D=\tau_0^2/K_{2r}$ is the characteristic dispersion length, $L_{NL}=1/(U_0^2\widetilde{W}_r)$ is the characteristic nonlinearity length, $R_\perp$ is the beam radius, and $U_0=(1/\tau_0)\sqrt{K_{2r}/\widetilde{W}_r}$ is the typical Rabi frequency of the probe field [32]. The dimensionless coefficients in Eq. (8) are given by $d_j=2L_D/L_j$ ($j=0-5$), where $L_0=1/K_{0i}$, $L_1=\tau_0^3\widetilde{W}_r/(\widetilde{\beta}_{1r}K_{2r})$, $L_2=-\tau_0^3\widetilde{W}_r/(\widetilde{\beta}_{2r}K_{2r})$, $L_3=6\tau_0^3/K_{3r}$, $L_4=\tau_0/K_{1i}$, and $L_5=(2\omega_p/c)R_\perp^2$ are the characteristic lengths of linear absorption, nonlinear dispersion, delay in nonlinear refractive index, third-order dispersion, and differential absorption and diffraction, respectively.

We note that if parameters of the system are chosen to make L_j ($j=0-5$) much larger than L_D, i.e., $d_j\ll 1$, the terms on the right-hand side (rhs) of Eq. (8) are high-order ones and can be taken as a perturbation. In this situation Eq. (8), when these perturbation contributions are neglected, reduced to the NLS equation given in Ref. [22].

In the present work, we consider an important situation where L_j are of the same order as L_D in addition to the requirement of the much smaller imaginary part of the coefficients of Eq. (6) comparing with the corresponding real part. In this case the terms on the rhs of Eq. (8) become significant and may not be treated as perturbations. One of the possibilities that can lead to such a situation is the case of shorter initial input pulse. Physically, reducing pulse duration while keeping probe pulse amplitude constant will lead to less effective third-order phase modulations. Note that soliton formation is strongly dependent on the effectiveness of these phase modulations which provide the necessary counterbalance effects to cancel the detrimental dispersion effects. Thus weakened phase modulation effects will lead to unbalanced dynamics, resulting in unstable propagation of ultraslow solitons. As we will show below, attention must be given to the significance of these higher order terms as the pulse duration is reduced.

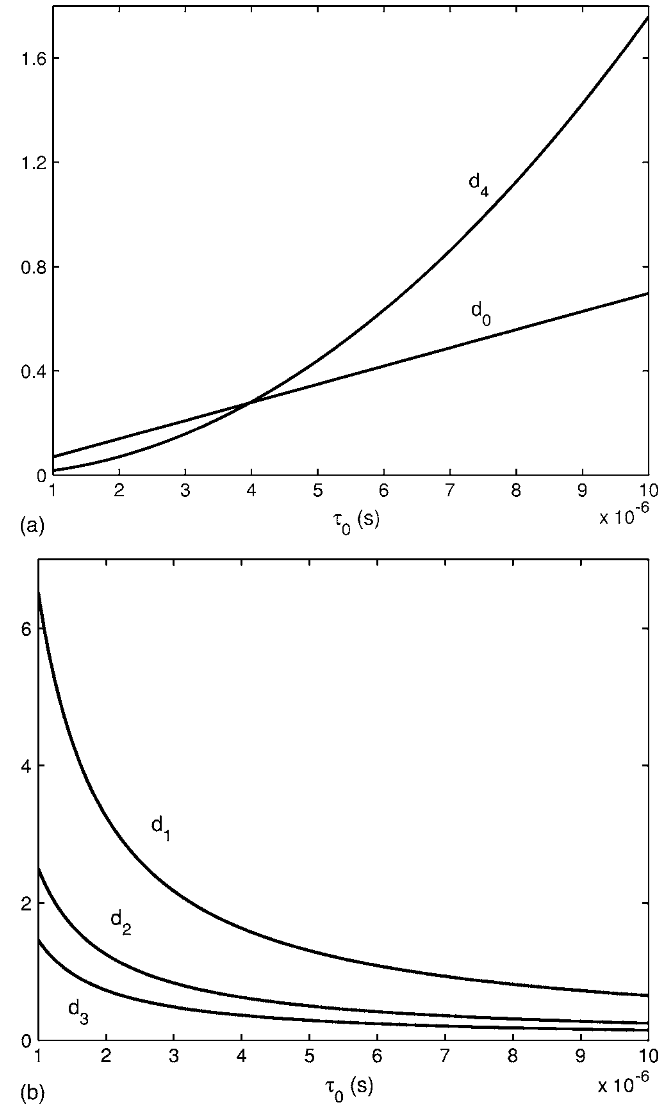

FIG. 2. The curves d_0 and d_4 (d_1, d_2, and d_3) vs pulse-width τ_0 in panel (a) [panel (b)].

To demonstrate the degree of importance of various terms on the rhs of Eq. (8) we consider the following set of experimentally relevant parameters suitable to a cold four-state atomic system: $\Gamma_2=2\gamma_2\simeq 2.0\times10^3\ \mathrm{s}^{-1}$, $\Gamma_3=2\gamma_3\simeq 1.2\times10^8\ \mathrm{s}^{-1}$, and $\Gamma_4=2\gamma_4\simeq 2.5\times10^8\ \mathrm{s}^{-1}$. We take $\kappa_{13}=1.0\times10^9\ \mathrm{cm}^{-1}\ \mathrm{s}^{-1}$, $2\Omega_B=3.2\times10^7\ \mathrm{s}^{-1}$, $2\Omega_C=6.0\times10^7\ \mathrm{s}^{-1}$, $\Delta_p\simeq-1.0\times10^9\ \mathrm{s}^{-1}$, $\Delta_B=1.2\times10^9\ \mathrm{s}^{-1}$ [34], $\lambda_p=c/\nu_p=0.8\times10^{-4}$ cm, and $R_\perp=0.1$ cm. In Fig. 2 we have plotted the coefficients d_j ($j=0-5$) as functions of pulse duration τ_0. From the figure we see that with the above set of parameters and for longer pulse duration, say $\tau_0\geq 1.0\times10^{-5}\ \mathrm{s}^{-1}$, the linear and differential absorptions, represented by d_0 and d_4, respectively, become relatively important. Correspondingly, the effects due to the nonlinear dispersion, delay in nonlinear refractive index, and third-order dispersion, represented by d_1, d_2, and d_3, respectively, become negligible. Thus, for pulse duration $\tau_0 \geq 10^{-5}$ s^{-1}, one has $|\Omega_{B,C}\tau_0| > 100$ and Eq. (3) is sufficiently accurate. However, if one reduces the probe pulse duration only, the relative importance of these two groups of effects will be reversed. This is clearly shown in the region of $\tau_0 \leq 10^{-6}$ s^{-1} where one has $|\Omega_{B,C}\tau_0| < 10$. In this region one must use the following generalized NLS equation

$$i\frac{\partial u}{\partial s} + \frac{\partial^2 u}{\partial \sigma^2} + 2|u|^2 u + i\left[d_1 \frac{\partial |u|^2 u}{\partial \sigma} + d_2 u \frac{\partial |u|^2}{\partial \sigma} + d_3 \frac{\partial^3 u}{\partial \sigma^3}\right] = 0 \tag{9}$$

to describe the evolution of the probe pulse. Note that all terms in the above equation have the same order of magnitude and thus must be treated on equal footing [35].

Equation (9) is well known in nonlinear fiber optics [33]. Various exact soliton solutions can be found using different techniques [36–39]. A single bright soliton solution is given by, after returning to original variables,

$$\Omega_p(z,t) = \frac{U_0}{\sqrt{2}d_3}\left(\frac{6(\beta + 3\Omega^2 - 2\Omega)}{3c_1 + c_2}\right)^{1/2} \times \text{sech}\left[\frac{\sqrt{\beta + 3\Omega^2 - 2\Omega}}{d_3 \tau_0}\left(t - \frac{z}{V_g^H}\right)\right]\exp[i\Phi(z,t)], \tag{10}$$

where $c_1 = d_1/(2d_3)$, $c_2 = d_2/(2d_3)$, $\Omega = (3c_1 + 2c_2 - 3)/[6(c_1 + c_2)]$, and β is a free real parameter relating with the soliton velocity. The phase function is given by $\Phi(z,t) = \Omega t/(d_3\tau_0) + \{K_r - \Omega/(d_3\tau_0 V_g) - [(\beta + 3\Omega^2 - 2\Omega)(1 - 3\Omega) + \Omega^3 - \Omega^2]/(2L_D d_3^2)\}z$. The conditions allowing the existence of the bright soliton solution [Eq. (10)] are $\Omega \neq 1/3$ and $\beta + 3\Omega^2 - 2\,\Omega$ being positive. By taking the numerical values of the parameters of the system given above, these conditions can easily be satisfied. Note that Eq. (10) is a nonperturbation solution of Eq. (9), i.e., it cannot be obtained using perturbation theory by taking d_j ($j = 1,2,3$) as small parameters.

The quantity V_g^H in Eq. (10) is the propagating velocity of the bright soliton, which is determined by

$$\frac{1}{V_g^H} = \frac{1}{V_g} - \frac{\beta \tau_0}{2L_D d_3}. \tag{11}$$

Choosing $\tau_0 = 3.2 \times 10^{-6}$ s and $\beta = 0.5$ we get $V_g^H/c = 3.6 \times 10^{-5}$. Thus the soliton shown in Eq. (10) indeed travels with very slow group velocity. This is an USOS based on the exact solution of the generalized NLS equation, Eq. (9). The average flux of energy over a period of the carrier-wave can be easily obtained. For the bright soliton given by Eq. (10) one has

$$\bar{P} = \bar{P}_{\max}\,\text{sech}^2\left[\frac{\sqrt{\beta + 3\Omega^2 - 2\Omega}}{d_3 \tau_0}\left(t - \frac{z}{V_{gr}^H}\right)\right], \tag{12}$$

where the peak power is given by

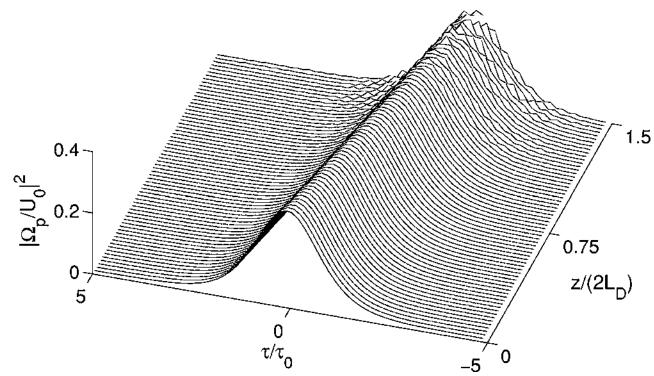

FIG. 3. The evolution of a single bright USOS in the presence of the linear and differential absorptions. The soliton solution (10) is taken as an initial condition in the computation. The soliton can propagate stably up to $z/(2L_D) = 1.5$.

$$\bar{P}_{\max} = \varepsilon_0 c n_p S_0 (\hbar/D_{31})^2 \frac{6K_{2r}(\beta + 3\Omega^2 - 2\Omega)}{d_3^2 \tau_0^2 \widetilde{W}_r (3c_1 + 2c_2)}, \tag{13}$$

where S_0 and n_p are the cross-section area and refractive index of the probe laser beam. Note that the peak power is directly proportional to the dispersion coefficient K_{2r} and inversely proportional to the square of the pulse duration τ_0 as well as the self-phase modulation coefficient $\widetilde{W}_r$. Using the above numerical example and taking $D_{31} = 1.2 \times 10^{-27}$ cm C and $S_0 = 1.0 \times 10^{-2}$ cm^2, we obtain $\bar{P}_{\max} = 3.6 \times 10^{-2}$ mW. Consequently, very low input power for the probe field is needed for generating the ultraslow optical soliton (10).

V. NUMERICAL SIMULATION

In order to test the stability of the USOS solutions of the generalized NLS Eq. (9) and to extend the analytical result given in the last section to include the influence due to the linear and differential absorptions, we have numerically investigated Eq. (8). In our simulations the soliton solution (10) is naturally taken as an initial condition. The space and time derivatives in Eq. (8) are performed by using a pseudospectral method (as used in Ref. [40]) and a fourth-order Runge-Kutta method for superior conservation of energy and other invariants, respectively. The parameters of the system are taken the same as those given above.

Shown in Fig. 3 is the evolution of a bright USOS in the presence of linear and differential absorptions, in addition to the higher order terms of Eq. (8) described there. In the figure we have plotted the relative intensity of the probe field (in terms of corresponding Rabi frequency), $|\Omega_p/U_0|^2$, as a function of τ/τ_0 and $z/(2L_D)$. We found that the linear absorption [represented by the term $-id_0 u$ in Eq. (8)] makes the soliton undergo a deformation, i.e., its amplitude (width) decreases (increases); while the differential absorption [represented by the term $d_4 \partial u/\partial \sigma$ in Eq. (8)] leads to an opposite effect. As a result, the detrimental effect due to $-id_0 u$ can be partially or completely canceled out, leading to stable soliton propagation up to $z/(2L_D) = 1.5$. Note that the dispersion

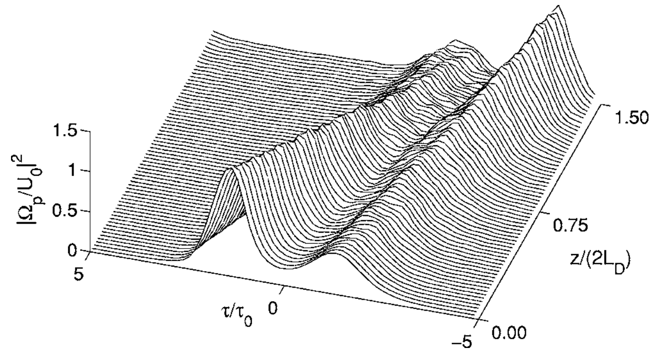

FIG. 4. The collision between two solitons is almost elastic and there appears a small phase (position) shift for each soliton after the collision.

length L_D, which is equal to the nonlinear length L_{NL}, of our system is 1.8 cm. Thus the soliton can propagate in the sample without apparent deformation up to $z=5.4$ cm. After this distance a small radiation from the soliton appears.

In Fig. 4 we have provided the numerical result on the interaction between two USOSs of shorter pulse duration with higher order terms included. The initial pulse used in integrating Eq. (8) consists of two USOSs with different amplitude and group velocity. We take $\tau_0=4.0\times10^{-6}\,s^{-1}$ and $\beta=0.5$ ($\beta=0.8$) in Eq. (10) and all other parameters are the same as those used in the simulation of the single USOS (Fig. 3). Thus the initial pulse reads $0.7\,\mathrm{sech}[1.2(\tau/\tau_0+2.0)]\exp[0.8(\tau/\tau_0+2.0)]+1.1\,\mathrm{sech}[2.0(\tau/\tau_0-2.0)]\exp[0.8(\tau/\tau_0-2.0)]$, where the separation is $T_0=4.0$. We can see that the collision between two solitons is almost elastic and there appears a small phase (position) shift for each soliton after the collision. As the relative phase in this case is approximately π, a repulsive interaction can be seen in the collision.

In addition, we have also investigated other interactions between two neighboring pulses with the higher order terms included. The initial pulse consists of two USOSs with equal amplitudes and the same group velocity. We take $\beta=0.5$ in Eq. (10) and all other parameters are the same as those used in the simulation of the single USOS (Fig. 3). We assume that the initial constant phase of the two USOSs are θ_1 and θ_2. Thus the initial pulse reads $0.6\,\mathrm{sech}[0.9(\tau/\tau_0+3.5)]\exp(0.6\tau/\tau_0+\theta_1)+0.6\,\mathrm{sech}[0.9(\tau/\tau_0-3.5)]\exp(0.6\tau/\tau_0+\theta_2)$, where the separation is $T_0=7.0$. Shown in Fig. 5 is the evolution plot of the neighboring USOSs with relative phase $\Delta\theta=\theta_1-\theta_2=0$ [panel (a)] and $\Delta\theta=\pi$ [panel (b)]. From the figure we can see that the interaction between the neighboring USOSs gives rise to unequal amplitude, which depends on the relative phase [shown in the insert figure of panel (b)]. However, the separation of the USOSs keeps almost constant, which is different from the case for the NLS equation [33]. Therefore we may infer that the combined effects of high-order dispersion and nonlinearity can restrict the interaction between the neighboring USOSs to some extent. This is advantageous for increasing the information bit rate in optical soliton communications [33].

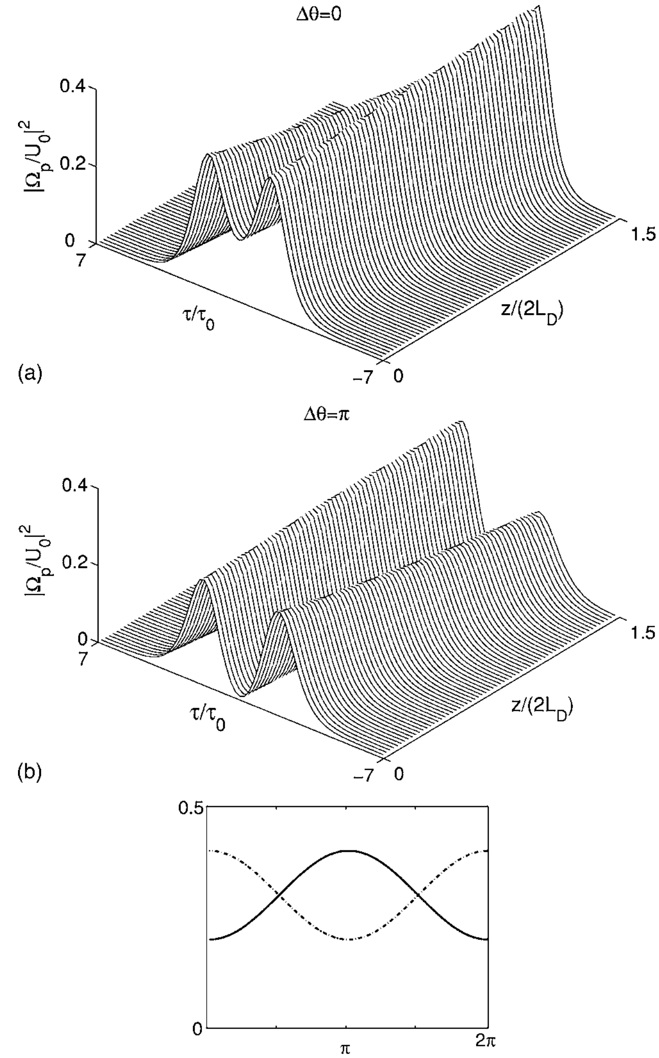

FIG. 5. The separating evolution plot of the neighboring USOSs with different relative phase $\Delta\theta=\theta_1-\theta_2$. Panel (a) gives the plot with $\Delta\theta=0$ while panel (b) gives the plot with $\Delta\theta=\pi$. The inset figure shows $|\Omega_p/U_0|^2$ vs $\Delta\theta$ at $z/(2L_D)=1.5$, the solid line represents the left soliton while the dotted line represents the right one.

VI. DISCUSSION AND CONCLUSION

As is well known, slowly varying envelope approximation (SVEA) has been widely used in the study of wave propagation in nonlinear optical media [41]. Based on this technique, many important physical processes, such as wave-wave resonant interaction, can be described in a very transparent way; and some interesting nonlinear localized phenomena, including spatial and temporal optical solitons, are predicted and found experimentally [33,41]. The SVEA is essentially a weak nonlinear and weak dispersion theory. It is applicable under the conditions of a weak light intensity and the power spectrum of optical field being concentrated in the neighborhood of some discrete frequencies. Thus one may use a perturbation expansion for the optical field and remove fast space and time variables, resulting in a considerable simplification for the problem under study. Usually, with this method one obtains a simple model in the form of one or several envelope equations. In recent years, the SVEA has been reformulated in a more standard and transparent way by using the method of multiscales [30,41,42].

It must be pointed out that any envelope equation obtained by the SVEA or the method of multiple-scales is valid only in some parameter regimes and for some space and time scales. This key point was illustrated clearly in Refs. [33,41,42] for optical pulse propagation in optical communications. Needless to say, the validity domains for the NLS model, i.e., Eq. (3) (see also Ref. [22]) and the generalized NLS model obtained in our present work, i.e., Eq. (6), are different. Using the parameters given in Sec. IV, the NLS model is valid for the pulse duration τ_0 around 1.0×10^{-5} s^{-1}. In this situation the nonlinear length L_{NL} and the dispersion length L_D are 18.0 cm [32]. The other lengths L_j ($j=1-3$) characterizing high-order dispersion, noninstantaneous Kerr nonlinearity are much larger than L_{NL} and L_D and hence can be safely neglected. In this sense the NLS equation is a suitable model for the solitonlike nonlinear propagation in the system. However, if τ_0 is less than 1.0×10^{-5} s^{-1}, the NLS model is broken down and one must include the effects due to the noninstantaneous Kerr nonlinearity and high-order dispersion. In our present work, different from Ref. [22], we have considered the nonlinear propagation for a shorter pulse with duration around 3.2×10^{-6} s^{-1}, and derived a modified NLS model, i.e., the generalized NLS Eq. (6), by employing the standard method of multiple-scales [30,41,42]. The noninstantaneous Kerr nonlinearity and the third-order dispersion of the system [43] have been included into the model in a standard and systematic way [44].

For a better confirmation of our approach, we have made an additional numerical simulation by starting directly from the MS equation (1). For a comparison between the NLS description, i.e., Eq. (3) (see also Ref. [22]), and the generalized NLS description, i.e., Eq. (6), we have chosen two different initial conditions. One of them is to use the single-soliton solution of the NLS Eq. (3), which has the form $U_0\,\mathrm{sech}[(t-z/V_{gr})/\tau_0]\exp\{i[K_{0r}-1/(2L_D)]z\}$. The other one is to use the single-soliton solution of the generalized NLS Eq. (6), given by Eq. (10). In both cases we take $\tau_0=3.2\times10^{-6}$ s^{-1} and other parameters are the same as those in Fig. 3. The results of the simulation are shown in panels (a) and (b) of Fig. 6, respectively. We see that an apparent deformation occurs for the NLS soliton [Fig. 6(a)] after propagating to $z=5.0$ cm. After that distance, the amplitude (width) of the soliton decreases (increases) and the soliton tends to split due to the effects of third-order dispersion and instantaneous Kerr nonlinearity. However, the soliton of the generalized NLS equation [Fig. 6(b)] is only affected by the linear and differential absorption and hence can propagate for a longer distance. These simulation results demonstrate again that the NLS equation (3) is not appropriate when the duration of the probe pulse is shortened to $\tau_0\approx10^{-6}$ or less. In this regime, one must use the generalized NLS equation (6) to describe a solitonlike pulse propagation in the system.

In conclusion, we have investigated the influence of high-order dispersion and nonlinearity on the propagation of ultraslow optical solitons in a lifetime broadened four-state atomic system under a Raman excitation. By employing a method of multiple-scales we have derived a generalized nonlinear Schrödinger equation and showed that for a realistic atomic system under suitable excitations the effects of third-order linear dispersion, nonlinear dispersion, and delay in nonlinear refractive index may be significant and therefore must be treated from a nonperturbative viewpoint. We have shown that the significance of these higher order terms become increasingly important when the pulse duration of the probe field is reduced. Based on the exact soliton solutions of the generalized NLS equation we have also demonstrated that it is possible to produce a new type of ultraslow optical soliton with group velocity on the order of $10^{-5}\,c$ in the four-state system. We note that such optical solitons are different from those found in Ref. [22] because the action to

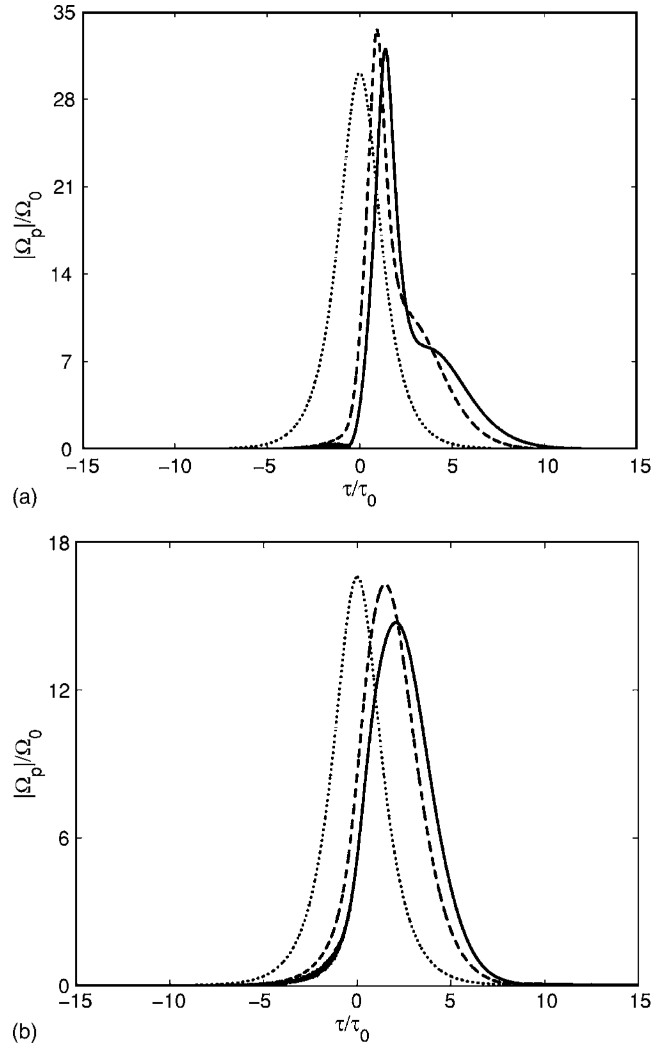

FIG. 6. The evolution of soliton for two different initial conditions. The probe pulse duration is taken as $\tau_0=3.2\times10^{-6}$ s^{-1} and the value of Ω_0 is 1.0×10^6 s^{-1}. In panel (a) [panel (b)], the initial condition is the single-soliton solution of the NLS equation (3) [the high-order NLS equation (6)]. The dotted line denotes the initial pulses, the dashed line denotes the pulses at $z=5.0$ cm, and the solid line represents the pulses at $z=8.0$ cm. We see that there is an apparent deformation for the NLS soliton [shown in panel (a)] while no apparent deformation for the soliton of the generalized NLS equation [shown in panel (b)].

balance the detrimental dispersion effects is due to quantities of different physical origins resulted from different nonadiabatic contributions. We have also carried out numerical simulations on the stability and interaction of ultraslow optical solitons with linear and differential absorptions being included. These numerical simulations show that formation and stable propagation of this new type of ultraslow optical solitons can be achieved in a four-state N type medium for an extended propagation distance.

ACKNOWLEDGMENTS

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APPENDIX: EXPRESSIONS OF THE COEFFICIENTS IN EQ. (7)

The expressions of the coefficients in the generalized NLS Eq. (7) are given by

$$K_{0i}=\frac{\kappa_{13}|\Omega_B|^2(|\Omega_B|^2\gamma_3+|\Omega_C|^2\gamma_4)}{\widetilde{D}^2}, \tag{A1}$$

$$K_{2r}=\frac{2\kappa_{13}}{\widetilde{D}}+\frac{2\kappa_{13}(|\Omega_B|^2\Delta_p+2|\Omega_B|^2\Delta_B+|\Omega_C|^2\Delta_B-\Delta_p\Delta_B^2)}{\widetilde{D}^2}-\frac{2\kappa_{13}|\Omega_B|^2(|\Omega_B|^2+|\Omega_C|^2-\Delta_p\Delta_B)^2}{\widetilde{D}^3}, \tag{A2}$$

$$\widetilde{W}_r=-\kappa_{13}\frac{|\Omega_B|^2[|\Omega_B|^4+|\Omega_C|^2(|\Omega_B|^2+\Delta_B^2)]}{\widetilde{D}^3}, \tag{A3}$$

$$K_{3r}=\frac{6\kappa_{13}(-2|\Omega_B|^2-|\Omega_C|^2+2\Delta_p\Delta_B+\Delta_B^2)}{\widetilde{D}^2}+\frac{6\kappa_{13}(|\Omega_B|^2+|\Omega_C|^2-\Delta_p\Delta_B)(-3|\Omega_B|^2\Delta_B-|\Omega_C|^2\Delta_B-2|\Omega_B|^2\Delta_p+\Delta_p\Delta_B^2)}{\widetilde{D}^3}+\frac{6\kappa_{13}|\Omega_B|^2(|\Omega_B|^2+|\Omega_C|^2-\Delta_p\Delta_B)^3}{\widetilde{D}^4}, \tag{A4}$$

$$\widetilde{\beta}_{1r}=-2\kappa_{13}\frac{|\Omega_B|^2}{\widetilde{D}}\widetilde{q}_1-\frac{|\Omega_B|^4+|\Omega_C|^2(\Delta_B^2+|\Omega_B|^2)}{|\Omega_B|^2\widetilde{D}}\widetilde{W}_r, \tag{A5}$$

$$\widetilde{\beta}_{2r}=-3\kappa_{13}\frac{|\Omega_B|^2}{\widetilde{D}}\widetilde{q}_1-\frac{|\Omega_B|^4+|\Omega_C|^2(\Delta_B^2+|\Omega_B|^2)}{2|\Omega_B|^2\widetilde{D}}\widetilde{W}_r, \tag{A6}$$

$$K_{1i}=\frac{2\kappa_{13}|\Omega_B|^2\Delta_B\gamma_3}{\widetilde{D}^2}+\frac{1}{\widetilde{D}^4}[2\kappa_{13}|\Omega_B|^2(|\Omega_B|^2+|\Omega_C|^2-\Delta_p\Delta_B)(|\Omega_B|^4\Delta_p\gamma_3+|\Omega_B\Omega_C|^2(\Delta_p\gamma_4+\Delta_B\gamma_3)+|\Omega_C|^4\Delta_B\gamma_4)], \tag{A7}$$

$$\widetilde{q}_1=\frac{|\Omega_B|^2+|\Omega_C|^2-\Delta_p\Delta_B}{\kappa_{13}\widetilde{D}}\left(\frac{1}{V_{gr}}-\frac{1}{c}\right)+\frac{(|\Omega_B|^2+|\Omega_C|^2)\Delta_B}{\widetilde{D}^2} \tag{A8}$$

with $\widetilde{D}=-|\Omega_B|^2\Delta_p-|\Omega_C|^2\Delta_B$ and $V_{gr}=c/[1+c\kappa_{13}(|\Omega_B|^4+|\Omega_B\Omega_C|^2+|\Omega_C|^2\Delta_B^2)/\widetilde{D}^2]$. Note that in order for simplifying the expressions the assumption $\gamma_2\ll\Omega_B,\Omega_C,\gamma_3,\gamma_4$ is used.

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[34] Large frequency detunings Δ_p and Δ_B are necessary in our paper. Typically, a 25% variation of these parameters does not result in appreciable changes.

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